STEP Examiner's Reports 2017

Mathematics

STEP 9465/9470/9475

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Contents

STEP Mathematics (9465, 9470, 9475)

STEP I 2017 Examiner's Report

The pure questions were again the most popular of the paper with questions 1 and 3 being attempted by almost all candidates. The least popular questions on the paper were questions 10, 11, 12 and 13 and a significant proportion of attempts at these were brief, attracting few or no marks. Candidates generally demonstrated a high level of competence when completing the standard processes and there were many good attempts made when questions required explanations to be given, particularly within the pure questions. A common feature of the stronger responses to questions was the inclusion of diagrams.

Question 1

This was a very popular question, attempted by 94% of the candidates. The substitution was often correctly made for the first part and a large number of candidates were able to identify the similar substitution required for the second integration.

The substitution for the first integration in the second part of the question caused some difficulties, but was again completed successfully by many candidates; however it sometimes required a number of attempts before the correct substitution was completed. A small proportion of the candidates were then able to complete the final integration successfully.

Question 2

This was the third most popular question on the paper, after questions 1 and 3.

Many candidates were able to show the result required in part (i) and to explain why the result also holds for the second range of values of x. However, a number of the solutions did not use integration as instructed by the question and so were not able to achieve all of the marks for the question. The second part was also carried out successfully by a large number of candidates and, as there was no specification that integration should be used for this part, a number of alternative solutions were also able to achieve full marks, providing that they started from the given inequality as instructed.

In the final part of the question most candidates were able to perform the required integration successfully and, while some were unable to follow through to reach the required final result, many did complete the question successfully and achieve full marks.

Question 3

This was another popular question, attempted by 94% of the candidates. The average mark achieved on this question was the highest of the paper and there were also more fully correct solutions for this question than any other on the paper. The majority of candidates appeared to be able to identify the steps that needed to be taken to complete this question and were able to perform the appropriate operations competently.

The final part of the question involved calculating the area of two triangles and many different valid approaches were seen. In many cases the successful solutions were accompanied by clear sketches, which enabled most of the remaining work to be completed successfully, or with only minor errors.

Question 4

This question was attempted by just over half of the candidates. The sketch required in the first part was generally very well done and most candidates were able to identify the appropriate features of their graph to explain the number of possible values for S. Many candidates were then also able to perform the appropriate substitutions to reach the required equation.

When tackling part (ii) many candidates were able to see how to apply the same line of reasoning to the amended situation. In many cases this was completed successfully, but a number of candidates failed to include the value of q that corresponds to the minimum point of the graph in the set of values that determine T uniquely. When trying to find the equation at the end of this part of the question some candidates struggled with rearranging to achieve a quadratic equation.

Question 5

This question attracted a relatively small number of attempts, many of which did not make very much progress and so did not score very well. As a result this question had the lowest average mark among the pure questions. Those candidates who did make some progress, however, often managed to produce quite good solutions and so there were still a number of attempts that were awarded full marks.

The majority of the successful attempts were accompanied by a clear diagram, which helped in understanding the situation as described in the question; candidates were then often able to follow through the steps as required.

Question 6

This question received the smallest number of attempts of all of the pure questions on the paper, a significant proportion of which did not go beyond an attempt to prove the first result before abandoning the question. The average mark for the question was therefore quite low. There were however a number of good responses to the question.

The proof by contradiction in the first part often received a partial explanation. Where the link between parts (i) and (ii) was seen candidates were often able to make good progress on the second part, although some errors in calculation occasionally led to incorrect examples of the function required.

Part (iii) required more care to work through successfully and only a small proportion of the candidates were able to see how to apply the previous result and then complete this part successfully.

Question 7

As with questions 5 and 6, question 7 attracted a small number of attempts compared to the other pure questions. It again received quite a low average mark, partly due to a large number of brief attempts which did not score any marks before the question was abandoned.

Diagrams again proved very useful in tackling this question and many candidates were able to solve part (i) correctly. The first equation to be shown in part (ii) was often reached accurately, providing that the relevant formulae were remembered correctly and many candidates were able to see how this led to the conclusion that the triangle is equilateral.

In part (iii) many candidates were able to show that the first condition implied the second, but there were some solutions that did not make it clear that the required implications worked in both directions for this part of the question.

Question 8

This question received a relatively high number of attempts, although many did not progress very far and so the average mark for this question was again quite low.

Many candidates were very competent with the process of proof by induction, although the fact that the question involved two related sequences caused difficulties for some. There were then a number of good solutions to part (i), but many did not manage to justify the limit of the sequence clearly enough to secure full marks.

The difficulty often encountered in the final part was in showing the first result. Often those who successfully achieved this were able to complete the rest of the question successfully.

Question 9

This was the most popular of the Mechanics questions, but still less popular than half of the Pure questions. Of all of the questions on the paper, this is the question that received the lowest average mark. Many attempts were able to produce the correct equations for the horizontal and vertical components of the motion. The differentiation required to then establish the result in part (i) proved quite complicated for many candidates and so many did not reach the required result.

Those who got as far as part (ii) were able to draw some conclusions about the relationships between the two angles, but struggled to reach the simplest form. Only a few candidates were able to achieve full marks for this question.

Question 10

Approximately one quarter of candidates attempted this question. In general the use of conservation of momentum and restitution was completed well by candidates, including in the case of the series of collisions. Part (i) was generally well answered, and many candidates were able to give at least a partial explanation of the result in part (ii).

Part (iii) caused considerably greater problems for many candidates, who struggled to identify the infinite series in order to evaluate the sum. Those who did successfully complete part (iii) were often able to complete part (iv) as well.

Question 11

While this question had the smallest number of attempts among the Mechanics questions, it did have the highest average mark. Many candidates were able to produce a diagram with the appropriate forces labelled and realised that the usual procedures of resolving in two directions and taking moments about a point would be a sensible approach. Despite the hint that taking moments about the midpoint of the rod might be helpful, a number of candidates chose to take moments about one of the ends of the rod, which led to more complicated sets of equations to solve.

The manipulation of the trigonometric terms proved challenging for many candidates, but a number did manage to work through to a clear and full solution to the problem.

Question 12

This was the second least popular question on the paper and many attempts only secured a small number of marks. Many of the candidates who attempted the question were able to form an appropriate expression for the expected profit, although a small number of solutions used the approximation too early and so did not give exact expressions at the points where they were required.

For the second part of the question the relationship between the three new variables was often found successfully and many of the candidates who attempted this part of the question were able to make progress towards the expected profit. A small number of candidates were able to follow through the final example to reach the required deduction.

Question 13

This was the least popular question on the paper and the only one where no candidate achieved full marks. Many candidates struggled to explain how the given situation could be described by the recurrence relations given. The elimination of t from the recurrence relations also proved problematic for many of the candidates. A few candidates were however able to show the solution for the sequence s and deduce the correct expression for the sequence t.

STEP II 2017 Examiner's Report

This year's paper was, perhaps, slightly more straightforward than usual, with more helpful guidance offered in some of the questions. Thus the mark required for a "1", a Distinction, was 80 (out of 120), around ten marks higher than that which would customarily be required to be awarded this grade. Nonetheless, a three-figure mark is still a considerable achievement and, of the 1330 candidates sitting the paper, there were 89 who achieved this. At the other end of the scale, there were over 350 who scored 40 or below, including almost 150 who failed to exceed a total score of 25.

As a general strategy for success in a STEP examination, candidates should be looking to find *four* "good" questions to work at (which may be chosen freely by the candidates from a total of 13 questions overall). It is unfortunately the case that so many low-scoring candidates flit from one question to another, barely starting each one before moving on. There needs to be a willingness to persevere with a question until a measure of understanding as to the nature of the question's purpose and direction begins to emerge. Many low-scoring candidates fail to deal with those parts of questions which cover routine mathematical processes - processes that should be standard for an A-level candidate. The significance of the "rule of four" is that four high-scoring questions (15-20 marks apiece) obtains you up to around the total of 70 that is usually required for a "1"; and with a couple of supporting starts to questions, such a total should not be beyond a good candidate who has prepared adequately.

This year, significantly more than 10% of candidates failed to score at least half marks on any one question; and, given that Q1 (and often Q2 also) is (are) specifically set to give all candidates the opportunity to secure some marks, this indicates that these candidates are giving up too easily.

Mathematics is about more than just getting to correct answers. It is about communicating clearly and precisely. Particularly with "show that" questions, candidates need to distinguish themselves from those who are just tracking back from given results. They should also be aware that convincing themselves is not sufficient, and if they are using a result from 3 pages earlier, they should make this clear in their working.

A few specifics:

In answers to mechanics questions, clarity of diagrams would have helped many students. If new variables or functions are introduced, it is important that students clearly define them.

One area which is very important in STEP but which was very poorly done is dealing with inequalities. Although a wide range of approaches such as perturbation theory were attempted, at STEP level having a good understanding of the basics – such as changing the inequality if multiplying by a negative number – is more than enough. In fact, candidates who used more advanced methods rarely succeeded.

Question 1

Almost all candidates attempted this question, making it the most popular on the paper; it was also the highest-scoring, with a mean score of 15. Indeed, the careful structure meant that its direction was clear to almost all candidates and it was only the rather tricky induction proof in (iii) that prevented the question from being completely transparent.

Question 2

This was the second most popular question of all, attempted by over 80% of candidates; but scoring relatively poorly with a mean score of under 10. It was, of course, heavily algebraic and this meant that many candidates found it challenging, getting lost in the algebra. In most cases, this was largely avoidable: the simple device of calling the first term "*X*" (say) would have prevented a lot of unnecessary subscripts from cluttering up the working. A few moments of thought from those candidates who simply embarked on the (potentially) intricate algebra could have saved a lot of trouble. The point of a sequence's periodicity is that it is the smallest cycle over which terms repeat; it should be noted that the condition for each term to be equal (a constant sequence) must clearly be embedded in any condition that gives $x_{n+2} = x_n$. Similarly, in order to satisfy $x_{n+4} = x_n$, we must automatically have the cases when all terms are the same *and* every other term equal present somewhere. This makes any ensuing factorisations much easier to deal with.

It could be noted that the requirement for $x_{n+4} = x_n$ can be thought of as a two-stage sequence using every other term; and this situation has just been sorted out.

Question 3

Attempts fell to around the 50% figure with marks scored by those who attempted the question averaging about 10 out of 20. There is not much to this question beyond the baseline realisation that $\sin y = \sin x$ does not necessarily imply that $y = x$. In essence, it is all about "quadrants" work, where candidates need to consider the two solutions, x and $\pi - x$ in one period of the sine function, and then adding or subtracting multiples of 2π as necessary. Once one has done this, the accompanying straight-line segments are straightforward marks in the last part of (i).

A lot of marks were gained in (ii), as candidates were clearly attracted by the familiar "differentiate this couple of times" demand; most of them were quite happy with the differentiation, performed either implicitly or directly using arcsines.
The drawings required in (ii) and (iii) then relied on an appreciation of the symmetries of the sine function, along with the use of the identity $\cos y \equiv \sin\left(\frac{1}{2}\pi - y\right)$.

Question 4

This is the first question where the difference between "attempts" and "serious attempts" arises to any significant extent: there were just over 800 of the former but well under 500 of the latter. This is also a good point at which to raise a key issue in respect of *strategy* for candidates sitting a STEP. Spending a few minutes of reading time, at some particular time during the examination, could be a significant asset, especially to those candidates who have particular strengths and weaknesses to play to or to avoid. A very brief analysis of this question, on first reading, should help one recognise that a result is being **given** (with no requirement to establish it in any way) and all that is required is to use it. Part (i) then clearly directs part of the way, and the required limits are rather obviously flagged, as is the fact that $g(x)$ must be something to do with the exponential function. One of the two functions to be used in (ii) is also given, as are the limits; an inspection of the **given** should lead to the (correct) conclusion that $g(x)$ *must* be $e^{-\frac{1}{4}x^2}$. Getting just this far takes the candidate to the 10-mark point, a perfectly good return for a candidate who has read the question through sufficiently carefully to realise that it has decent potential for mark-acquisition.

In the final part of the question some careful thought was needed, with only the required limits obvious at first glance. Most attempts, serious or otherwise, picked up the majority of their marks in (i) and (ii) and efforts at (iii) were very varied: many candidates simply gave up and moved on; many more picked up a few extra marks by setting $g(x) = \sqrt{\sin x}$ (which is a fairly obvious candidate to try) and working towards the right-hand half of the given result. Very few candidates indeed had the experience to realise that $\sqrt{\sin x}$ now needed to appear as the squared term, which also meant that a cosine term had to be involved.

Question 5

Attempts at this question were over the thousand figure, making it the third most popular question on the paper, with the second-highest mean score. Part (i) proved to be very routine; the calculus requirements in (ii) were obvious to most, though justifying the minimum distance was often poorly handled; for instance, finding the second derivative is a poor way to spend one's time when examining the sign of the first derivative is easily undertaken. The needs of part (iii) were also easily spotted though, again, a couple of marks were almost universally lost as the need to eliminate the two other cases that arise was largely ignored.

Question 6

This question was relatively popular, but it turned out to be one of the hardest of the pure questions. The first part was a reasonably standard example of induction but nearly all candidates failed to understand the subtlety of what was required in the last part.
Most candidates made significant progress with part (i), clearly being familiar with the process of induction. However, the algebra to complete the proof was too much for most candidates. Inequalities were frequently handled poorly and the general presentation of logical arguments was unclear with many candidates assuming what was required and not making implications clear. Attempts which brought in calculus were rarely relevant and even less frequently correct.

In the second part, candidates tended to overcomplicate the question. Squaring up (since both sides are clearly positive) and expanding brackets was all that was required. It would have been nice if students had shown some awareness that the squaring process was valid since both sides were positive, however if we had required this it would have effectively been a one mark penalty for all candidates.

In the final part candidates often considered S_1 to find a necessary bound on C. However, further work – usually an induction using their guess – was required to show that this bound works for all n. Many candidates seemed unaware that this final stage was required.

Question 7

Just over one thousand candidates attempted this question, but more than 400 of these attempts were not substantial; removing the large number of those scripts which got no further than part (i) raises the mean score from well under 8 to just over 12 out of 20.
The difficulty with questions like this is that it is very easy to make correct statements but much more difficult to support them with logically-crafted steps of reasoning based on results either given or known. Moreover, one needs to reason in such a way that the steps of working one writes down are justified … this was the principal barrier to anything more than the most faltering of starts. So, part (i) was an issue for candidates, with much written but not much of it coherently stated or supported. Of the few marks gained in the weaker attempts, part (ii) provided the majority of them, since most candidates were happy to take logs and then differentiate (the standard procedure for exponential equations of this kind).
It was slightly surprising to note that so few candidates attempted to establish the initial result in part (iv) using calculus; most of those that got this far presumably thought some other "inequality" technique was being tested.
Finally, even for those who had made good progress in several of the previous parts, the graphs at the end were frequently marred by a lack of labelling.

Question 8

The vectors question again proved extremely unpopular, despite the fact that it is perhaps the easiest question on the paper. It drew the least number of attempts from the Pure Maths questions (the only one under half of the entry) and two-thirds of these were not substantial attempts. In this case, it is easy to say what (almost invariably) appeared: candidates generally got no further than the first three marks, which could be gained by writing down two line equations, $\mathbf{r} = \mathbf{a} + \lambda\mathbf{u}$ and $\mathbf{r} = \mathbf{b} + \mu\,\mathbf{v}$, and then equating the two expressions for $\mathbf{r}$. Few made further progress, revealing a reluctance to engage with the algebraic manipulation of vectors (handling numerical vectors is, of course, a completely different matter altogether).

Question 9

This question was the least popular of the mechanics questions. Even amongst those who attempted this question only about a quarter made any meaningful attempt. As in so many mechanics questions a good, clearly labelled diagram often meant the difference between candidates making no progress and good attempts. It does seem that many students are reluctant to attempt these types of problems. It is hoped that looking at the hints and solutions should help candidates see that some resolving and taking of moments often leads to efficient solutions of problems like this.

Question 10

This was another very unpopular mechanics question. Many candidates who might have thought they made considerable progress did not score very well because they failed to communicate clearly the details required to “show” the given results.

Question 11

This was by far the most popular and most successfully answered mechanics question. The general concepts of energy and kinematics certainly seem to be familiar to most candidates, although there was a certain amount of “throwing SUVAT equations around” without any particular strategy, hoping something would miraculously appear.

The final part proved much more challenging. Several candidates attempted symmetry arguments, but these lacked the required rigour.

Question 12

The statistics questions were attempted by only a small fraction of the cohort, with question 12 the least popular question on the paper, receiving fewer than 200 attempts, of which only about a third made any meaningful progress. Although the question had a small wording ambiguity this did not seem to have bothered any but a very few candidates. It was disappointing that even the fairly standard analysis in part (i) proved difficult for most candidates, with several claiming that the mean and the variance being equal was a sufficient condition for $X + Y$ to follow a Poisson distribution.

Question 13

The first two parts of this question were reasonably straightforward, but this was only marginally more popular than question 12. A surprising number of candidates did not seem to be confident dealing with telescoping fractions – a fairly common tool in probability. The algebraic demands of the final part proved challenging for many candidates.

STEP III 2017 Examiner's Report

The total entry was only very slightly smaller than that of 2016, which was a record entry, but was still over 10% more than 2015. No question was attempted by in excess of 90%, although two were very popular and also five others were attempted by 60% or more. No question was generally avoided with even the least popular one attracting more than 10% of the entry. Less than 10% of candidates attempted more than 7 questions, and, apart from 18 exceptions, those doing so did not achieve very good totals and seemed to be 'casting around' to find things they could do: the 18 exceptions were very strong candidates who were generally achieving close to full marks on all the questions they attempted. The general trend was that those with six attempts fared better than those with more than six.

Question 1

The most popular question on the paper, attempted by about 84% of the candidates, it was also the most successfully answered with an average mark of about 12/20. The first result was generally well answered with a few candidates attempting to use induction, and then proving the result directly. The summations were usually done well, though often lacked explanation. Usually, the inequalities were not well argued, there was poor layout, and no mention of positivity. Those who spotted the link with part (i) did well in general summing the inequalities, though there were some problems with the indices.

Question 2

This was the least popular pure question being attempted by only just over a quarter of candidates, and was the least successful of all the questions scoring 5/20. Most candidates gave up after part (i), and some made much more of this first result, not being very succinct. Most could write down SR without difficulty, but then did not spot an easy way to move beyond this. The standard of algebra displayed was in general poor, in particular moving between complex and trigonometric forms.

Question 3

The second most popular question at just over 70%, the success rate was about half marks in common with a number of other questions, with the majority earning either 16 and above, or 4 and below. A common mistake was omitting the minus sign in the first step to obtain A which resulted in candidates being unable to progress further. If the cubic equation was correctly found, then candidates tended to score all the marks as far as part (iii). A few candidates obtaining the correct results in (iii) then stated that the answers could not be complex, which was, of course, false.

Question 4

Three fifths of the candidates attempted question 4 with a marginally better success rate than question 3. A significant proportion of candidates struggled with changing base for part (i), but almost all completed (ii) successfully. A common strategy for part (iii) was to use the result of part (i) but very few remembered to check for $b = 1$. There were very few successful attempts for part (iv); many tried integration by parts, but rarely successfully.

Question 5

Very slightly more popular than question 4, the marks scored were on average 1 less per attempt. Most found $\frac{\mathrm{d}y}{\mathrm{d}x}$ successfully, though a significant minority swapped x and y. In this case, they could still obtain the displayed equation successfully, but in both categories, there were frequent sign errors when differentiating trigonometric functions. Most then attempted using the displayed result to find $\mathrm{f}(\theta)$, either by separating variables or using an integrating factor and got as far as $\mathrm{f}(\theta) = \left(\frac{k\cos^2\theta}{1+\sin\theta}\right)$ but then more than half got stuck. Most plotted the two given curves relatively correctly, but then a substantial number used guesswork having not previously obtained C correctly.

Question 6

The second most popular question attempted by four fifths of the candidates; the success rate was only very slightly less than that of question 4. As every part of the question required obtaining a given result, it had to be marked strictly on how well things were presented. There were surprising problems with changing the variables in the first part, as often candidates did not clearly understand dummy variables, and others integrated with respect to constants. In spite of the ban on the use of trigonometric functions, some still tried to use the tangent function. The two results in (iii), especially the second, were testing but were found very hard, and previous inapplicable results were used, ignoring the conditions given as inequalities.

Question 7

With popularity between that of questions 4 and 5, the mean score was about 8/20, making it one of the least successful pure questions. Most candidates attempting this question did the stem correctly and then scored about half the marks on (i) before stopping, either due to mistakes in the gradient computation or commonly not identifying the $(1+t^2)^2$ in the constant term of the line equation. A common slip was to differentiate $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ implicitly as $\frac{2x}{a^2}+\frac{2y\frac{\mathrm{d}y}{\mathrm{d}x}}{b^2}=1$. The geometric interpretation in (i) was frequently omitted, and there were numerous and varied incorrect suggestions for the case $X^2=a^2$. Few continued to (ii), though it should be observed that some courted disaster by labelling the coefficients of the quadratic in (i) as a, b, and c.

Question 8

This was attempted as many times as question 5, but the success rate was about halfway between that of questions 5 and 7. Many attempts were made using induction which wasted a lot of time. Otherwise, in general, the stem and part (i) were well solved, but many could not spot the method to proceed with part (ii).

Question 9

The most popular of the applied questions, there were a handful of attempts more for this question than for question 2. However, it was the least successfully attempted applied question with about one third marks scored. Common errors were to assume constant acceleration which does not apply, or to consider the motion of the centre of mass, but ignoring the normal force at the edge of the table, and the fact that the centre of mass does not lie along the string once motion commences. The two constants of integration for the first result were in fact zero but needed to be shown to be so. In considering the energy of the system, many assumed the speeds of A and B were equal, which they do work out to be, but this could not be known before calculating correctly, and likewise the elastic energy being zero, which again needed to be shown. Numerous attempts resulted in the given correct speed from specious working. Scoring largely occurred in the first section of the solution, though rarely earning all the marks for that first result and then earning little attempting to conserve energy.

Question 10

Attempted by about one eighth of the candidates, the success rate was only marginally better than that for question 9. The first displayed result and the expression for $\ddot{\theta}$ were generally successfully dealt with by those candidates who knew how to apply moments of inertia. After that point, most mistakes were either algebraic or incorrect signs in the equations derived by resolving forces to obtain acceleration. About half of those that reached the end of the question correctly interpreted the physical meaning of the case $\ell > 2a$. However, a common misinterpretation was that the particle would begin to slip at this point.

Question 11

A fifth of the candidates attempted this with just a little less success than that for question 5. Only a minority drew a sketch of the problem; had more done so, some errors might have been precluded. In part (i), a large minority ignored the condition 'initially at rest', a handful gave a negative answer for u, and a few attempted to conserve energy, but that was rare. In the second part, some candidates attempted to just write down the given answer employing conserved momentum with verbal justification. The inequality generally followed if a telescoping argument was used although the care shown in dealing with the strict inequality and the $r = n$ case was poor. Attempts at (iii) were generally sound though some took the projectile speed as u.

Question 12

The least popular question on the paper, it was still attempted by just over 10% of the entry achieving marks only very marginally less good than for question 3. It was fairly well done overall, though a few were completely confused, so the marks tended to either be very high, very low or about around half marks for some who did not do much on part (ii).

Question 13

A sixth of the candidates tried this, scoring slightly less well than was done on question 11. Almost everyone found $\mathrm{V}(x)$ correctly and the required result for $\mathrm{E}(Y)$. Similar success was demonstrated finding $\mathrm{V}(x)$ in the uniform case. A lot did not then attempt to find the probability density function, but most who spotted it attempting to calculate the cumulative distribution function of Y first and then differentiate could do it. It was encouraging that so many correctly found the range of Y. A variety of methods of integration were used for the final result with varying success.

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